Investigation of the Interacting Flow of Nonsymmetric Jets in Crossflow

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Asymmetric jets in crossflow provide complex interacting flowfields that contain many vortices. These vortices change significantly with variations of the jet and the crossflow and under certain specific conditions in which shedding vortices appear and disappear periodically. Four different cross-section geometry jets were studied and compared with a circular cross-section jet, all with the same jet exit area. Various flow visualization techniques were used to aid in understanding the highly complex three-dimensional flowfield. Among many forms of secondary vortices observed, in general, four major vortices were identified. The most notable and interesting group was the "spin-off" vortices observed previously in our study of discrete jets blowing from a wing tip. These shedding vortices were formed under certain flow conditions and jet orientations. The shedding vortices were nearly parallel to the jet exit axis, with periodicity depending on the jet/freestream velocity ratio. A schematic reconstruction was made of the asymmetric flowfield based on observations of the flow evolution, and relationships with symmetric jets in crossflow were established. Unsteady jets created by pulsation of the jet flow at low pulsation frequencies were observed to behave quite differently from the steady jets and penetrated farther into the crossflow.

I. Introduction

JETS in a crossflow are of practical interest in many engineering fields. From the VSTOL aircraft in a transition flight to the waste disposal into bodies of water or the atmosphere, the characteristics of jets in crossflow play a dominating role. Consequently, a large volume of literature has been devoted to this topic (see, e.g., a survey by Rajaratnam[1] in 1976 and many subsequent papers in various journals). In spite of such extensive investigations on the subject of jets in crossflow, we have encountered the asymmetric jet problem that, to our knowledge, has not received proper attention anywhere. The motivation for this study stems from our previous investigation,[2-4] where discrete jets, blowing from the wing tip, were used in order to improve the wing aerodynamic performance. Discrete jets applied at a wing tip on a rectangular wing resulted in significant lift improvements with small jet blowing momentum coefficients. It was observed that the entire wing circulation had been altered.[3,4] This was evidently the result of interaction of flow around the wing with the wing tip jets. The wing tip jets were blown from rectangular exit cross sections. The major axis of rectangular jet ports was placed parallel to the wing chord line, so that the individual jet ports were inclined to the local oncoming freestream. Therefore, asymmetric jet flows were believed to be essential for understanding the whole flowfield of the wing tip jets. The peculiar behavior of auxiliary vortices was observed (Fig. 1). It was suspected that the nonsymmetric jet in crossflow created by yawing a symmetric elongated jet was responsible for the generation of these vortices. Consequently, the study was undertaken to investigate the flowfield of asymmetric jets in uniform crossflow.

Undoubtedly, such asymmetric flowfields may be encountered in many actual occurrences in nature or in the engineering applications. Take, as a simple example, the situation of a vertical jet takeoff aircraft, with the jet blown from noncircular jet ports (say, rectangular), encountering a crosswind that is not aligned to either of the major axes. With the courtesy of Prof. G. Carrier of Harvard University, we obtained a photograph showing an oil rig on fire in a strong wind over the ocean (Fig. 2). Judging by the size of the nearby ship, the Reynolds number involved in this case must be very large. The jet (smoke) plume with crosswind induced many vortices that were very similar to what we have observed in the water-tunnel experiments on asymmetric jets (see typical flowfield shown in Fig. 3). Therefore, as is apparent from this comparison, the phenomenon observed in this study is not limited to moderately small Reynolds numbers.

Many investigations have been performed on circular jets in crossflow and limited studies made of rectangular jets symmetrically aligned with oncoming flow.[5-9] In the past, vortices shedding from such flow phenomena have been studied in detail. Some of the observations are relevant to the present asymmetric jet study. Moreover, the von Kármán vortices and wake instability of flow over a cylinder or bluff bodies are also relevant to the problem.

The present study concentrates on detailed features associated with the nonsymmetric jets in crossflow, features that could be enhanced by proper geometrical modification or introduction of periodic excitations to aid the wing tip jet flow interaction effects. Recent papers by Wu[10] and Wu et al.[11] indicate that the wing tip vorticity distribution has a significant impact on the wing performance.

II. Experimental Procedure

At the onset, the basic goal of this study was to understand fundamental flow phenomena that may lead to design jet shapes whose flowfield would resemble some of the key flow features observed in the wing tip jet flow study, as described in the Introduction. In order to avoid the added complexities of the near wing tip flowfield, the problem has been simplified to nonsymmetric jets blown perpendicularly from a *flat plate* into uniform crossflow.

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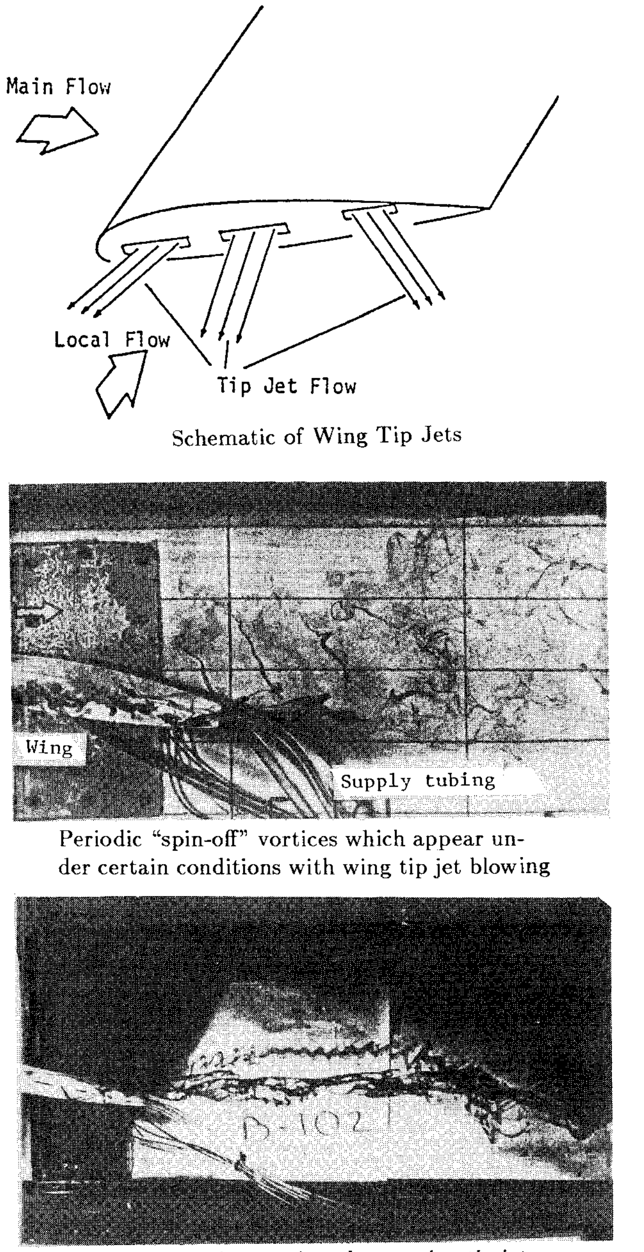


Schematic of Wing Tip Jets

Periodic "spin-off" vortices which appear under certain conditions with wing tip jet blowing

Counter rotating vortices due to wing tip jets

Fig. 1 Discrete wing tip jets and selected observations (from Ref. 1).

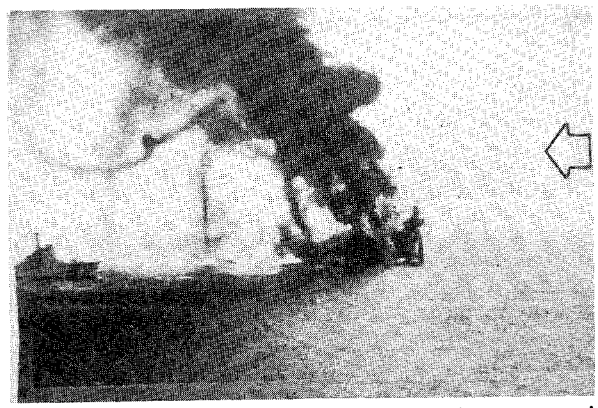


Fig. 2 Offshore oil rig in flames in crosswind (private communication with G. Carrier).

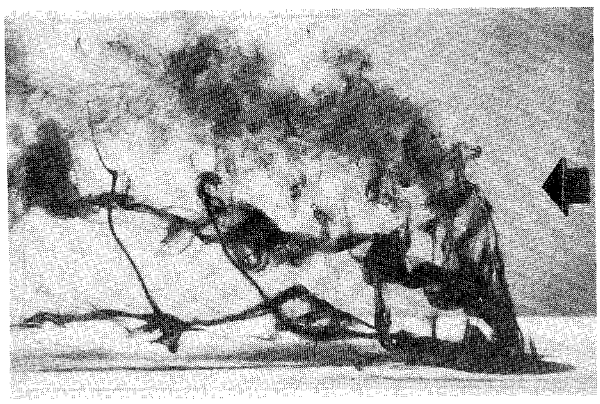


Fig. 3 Nonsymmetric jet in crossflow (present study).

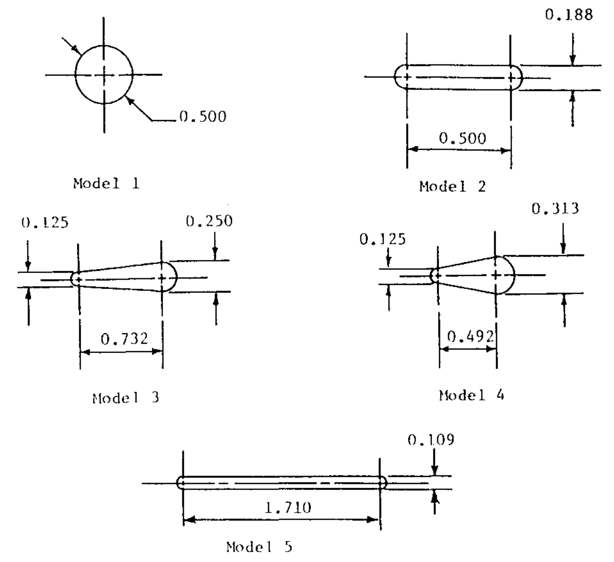
Five jet ports with different cross-section geometry but with equivalent cross-sectional area were designed and manufactured. Figure 4 shows the dimensions and coordinates for these jet ports. These five shapes are numbered and named for ease of reference. The model 1 jet, which is the circular jet, is the basic reference geometry. A large number of studies and measurements are available for the circular jet in crossflow; hence, it is naturally a good geometry to be used for comparison with observations on the other four jets.

Each jet was installed on the water-tunnel floor on a modular common base extending into the tunnel 1.9 cm (0.75 in.), where the jet exit was flush-mounted onto a boundary-layer splitter plate. The splitter plate was 150 cm (60 in.) long and extended 19 cm (7.5 in.) upstream of the jet port center to allow for the development of the flat-plate laminar boundary layer. The modular components mechanism allowed adjustments to be made in the orientation of the various jets only in the plane of jet exit with respect to the uniform crossflow. Therefore, the jets were always perpendicular to their exit plane or to the tunnel floor but could be easily rotated to create yaw angles of $\beta = 0$–180 deg (Fig. 4b).

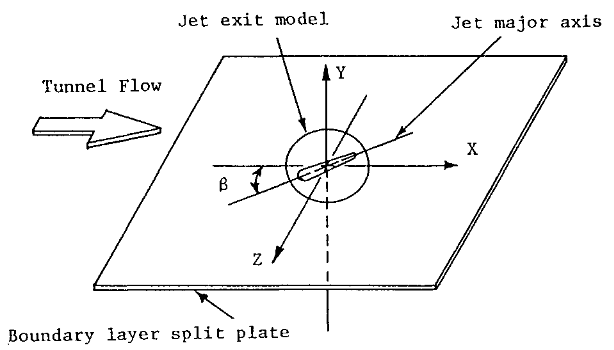
The University of Texas Space Institute water tunnel is a closed-circuit, continuous-flow facility designed especially for low-turbulence and high-quality flow visualization. The tunnel lies in a horizontal plane and is powered by a twin-bladed propeller in the return leg of the tunnel circuit. The tunnel freestream speed is varied from 1.5 to 60 cm/s by changing the propeller rotational speed. The Reynolds number based on the jet diameter was in the range of 200–500. Test section dimensions are 35.6 cm high, 45.7 cm wide, and 150 cm long ($15 \times 18 \times 60$ in.). The test section walls are made of Plexiglas for versatility in observing and photographing the flowfield. The experimental setup is schematically shown in Fig. 5.

Two types of flow visualization techniques have been used to extract maximum information from the flowfield. The first was by using the regular milk and dye technique. The dye was a solution of alcohol, milk, and commerical food coloring, mixed in such a way as to insure specific gravity equivalent to water. The second type was the laser-induced flourescent technique. Three chemicals were used: rhodamine 6G, rhodamine B, and fluorescein sodium salt. Once dissolved in water and under the excitation of argon-ion laser light, yellow, red, and green fluorescent colors were visible. Dye was injected into the specific areas of the model at controlled rates desired for each dye exit port. Many distributed small dye exit ports were built into the plate and jet exit neighborhood so that the dye could be introduced at the desired points on the surface. Small dye probes, capable of being positioned inside the flow, were also used to introduce dye at regions of flow to which the surface dye was not carried.

An argon-ion laser light source (1–2 W power) was used for the laser fluorescent flow visualization. A 4-mm cylindrical lens was used to form a planar light sheet. Combined translation and rotation of the light sheet plane provide visibility into the complete flowfield at any desired orientation. Frequently, the dye for laser flow visualization was introduced into the jet



a) Geometry of five jet exit models (all dimensions in inches)



b) Coordinates and jet exit model orientation

Fig. 4 Schematic of experimental arrangement.

fluid itself. In this way, it became possible to observe directly the jet fluid deformation, which was not otherwise visible. On the whole, the two flow visualization techniques complemented each other for a complete observation of the details.

In order to study different jet entrainment and interaction effects, the tunnel velocity and the area-averaged jet exit velocity was chosen such that the velocity ratio was normally $1 \leq V_j/V_\infty \leq 9$. For each jet, the major axis of jet exit model was adjusted and the relative yaw angle β was varied to enable study of the vortical flow interaction effects. Pulsation of a jet at frequencies between 1 and 16 Hz were applied. A small, oscillating rectangular fin in the wake of a jet was used to amplify the periodic vortical flow interaction effects. Various observations and the resulting analysis of the flow visualization study are discussed in the following sections.

Velocity measurement of vortical flowfield was performed by a three-component hot-film anemometer. This system is composed of a customized TSI three-component hot-film probe, a TSI model IFA 100 Intelligent Flow Analyzer, Scientific Solution's Lab Master 16 channel analog-to-digital converter, and an IBM PC/XT compatible computer with 20-MB hard disk. The three-component hot-film probe and the measurement details are discussed in Ref. 22.

III. Results and Discussion

A. Main Jet Induced Vortices

Jets in uniform crossflow are acted on by the momentum of the cross stream from the point of exit until complete balance

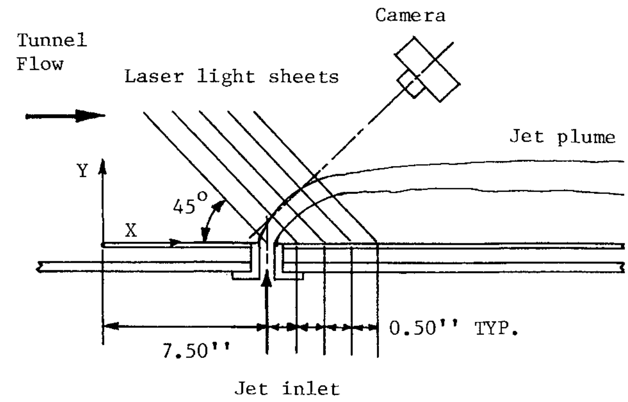


Fig. 5 Arrangement for laser-induced fluorescent flow visualization and sectional view of jet plume.

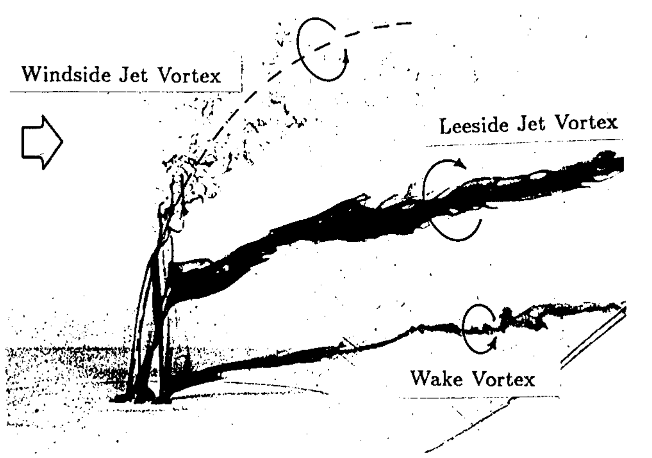


Fig. 6 Flow in the downstream of jet model 4, indicating various steady vortices; $V_j/V_\infty = 3.7$, $\beta = 30$ deg, $Re_d = 240$.

and alignment between the two flows are reached. Because of the jet bending and curvature, a pressure gradient is generated in planes perpendicular to the centerline of the jet flow. The jet fluid and the entrained crossflow form two distinct vortices (called Prandtl's secondary flows of the first kind[12]). For a symmetric jet in a uniform cross stream, these two vortices are symmetrically generated, with the jet wake also symmetric. Such observations have also been made in many other studies, as noted earlier.

If the jet forms a nonsymmetric configuration with the crossflow, the resulting flowfield around the jet and in the downstream is highly asymmetric, three-dimensional, and usually unsteady. The various vortices are formed, and their interactions with the wake flow produces a highly complex flowfield. Under these conditions, the jet vortices are not symmetric, and their strengths are different. Also, the jet fluid is acted on nonuniformly by the crossflow momentum; hence, the two jet vortices follow two different trajectories. All these introduce a significantly different flowfield compared to the symmetric jets.

By introducing colored dye from appropriate locations near the jet exit, it was possible to make the core regions of the vortices visible. The core traces also correspond to their mean trajectories. A typical illustration of the major vortex trajectories for model 4 can be seen in Fig. 6. This side-view photo was taken with a moderately high jet/freestream velocity ratio of 3.70, with the short teardrop-shaped jet port (model 4) yawing at $\beta = 30$ deg relative to the oncoming freestream. Three distinct major vortex systems could be identified in the above flow and the jet orientation conditions. The most penetrated wind-side jet vortex appeared to be the strongest one, and with regular dye usually the least visible in the downstream because

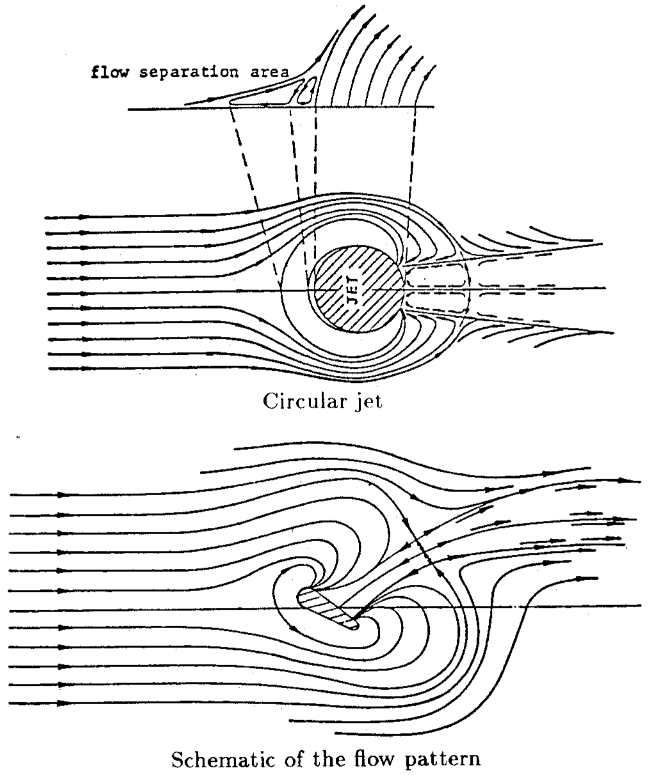


Fig. 7 Surface dye flow pattern around jet model 3; $V_j/V_\infty = 9.0$, $\beta = 31$ deg, $Re_d = 240$.

of strong entrainment of the crossflow and locally higher turbulence level. With the laser-flourescent flow visualization, however, this vortex flow became very distinct. (The sense of rotation of these various vortices are discussed in Fig. 9.)

For small jet/freestream velocity ratios, all three vortices were lying close to the flat plate and less distinguishable because they were closer, weaker, and nearly merged together. For small yawing angles β, the jet vortices represented by the upper two trajectories followed similar paths and, from the side view, appeared to coincide. This was expected because the jet vortices appeared to be almost symmetric for decreasing yaw angle β. For increased jet/freestream velocity ratio, the jet vortices represented by the upper two trajectories penetrated farther away into the freestream. In the meantime, it appeared that an additional pair of less distinct vortices was formed from the jet exit to join the major (stronger) wind-side jet-vortex flow. In other words, there is a possibility of forming more major vortex systems with a higher velocity asymmetric jet in crossflow.

Flow on the plate around the circular jet and the teardrop asymmetric jet is shown in Fig. 7. Similarly to the horseshoe vortices in front of a solid cylinder in crossflow, a separation region also exists in the flow just upstream of the various jets. For the circular jet, the flow pattern is schematically shown in the top of Fig. 7. The horseshoe vortices and the flow around the jet exit on the plate are symmetric for a symmetric jet exit. For an asymmetric jet exit with respect to the crossflow, the flow pattern is not symmetric even though the basic flow physics are preserved. The surface flow pattern in the downstream of the teardrop-shaped jet (model 3) is shown in a top-view illustration in Fig. 7. Even though the jet is issued perpendicularly to the surface, the surface wake is dominated by the nonuniform entrainment and is therefore nonsymmetric. The surface shear stress lines, visualized by placing dye upstream on the plate around the jet, are used to help the interpretation of flowfield near the flat-plate surface. The flowfield near the plate resembles that of a wing at an incidence to a uniform flow.

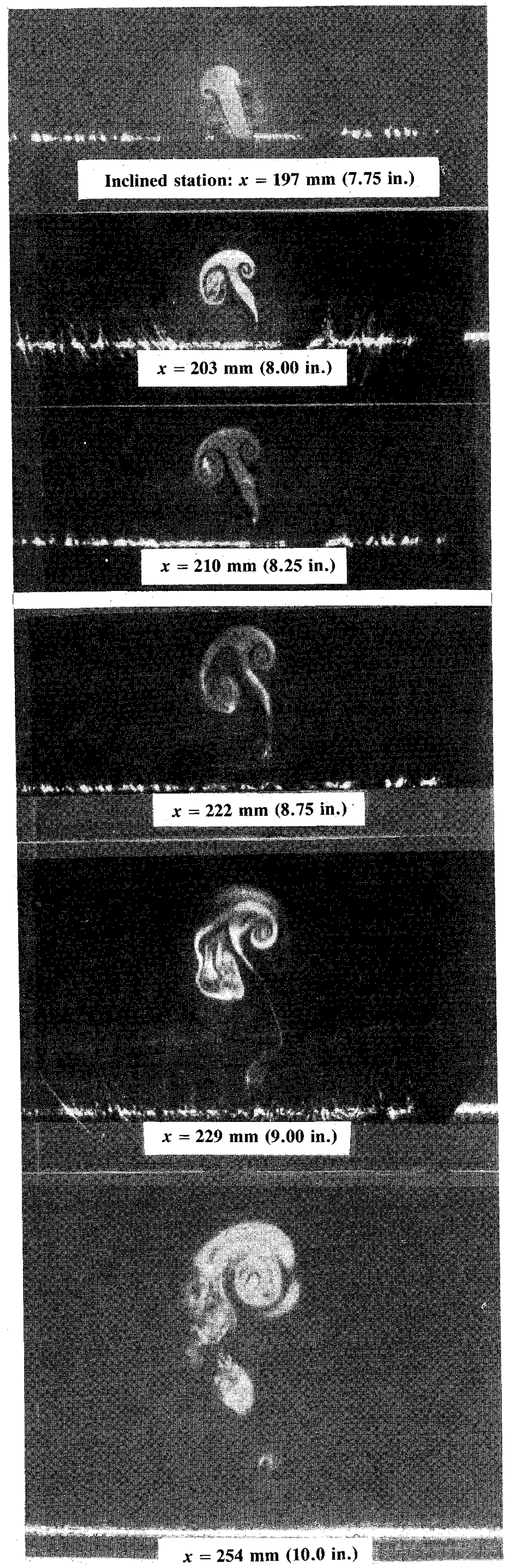


Fig. 8 Sectional view of the evolution of jet stream model 3 in crossflow on 45 deg inclined planes at discrete downstream stations; $V_j/V_\infty = 2.0$, $\beta = 20$ deg, $Re_d = 470$.

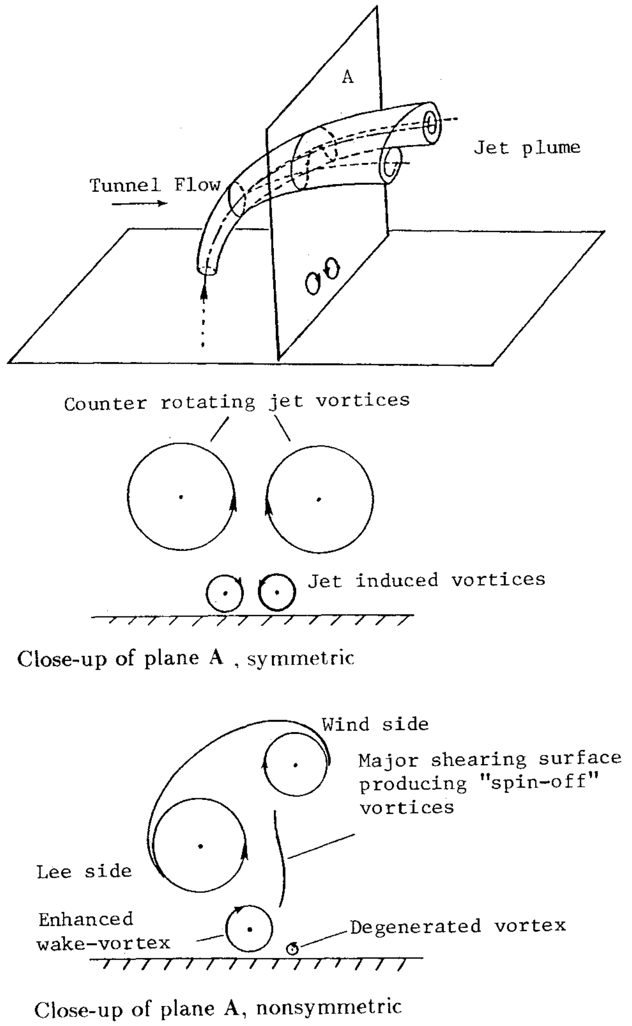


Fig. 9 Comparison of vortices for symmetric and nonsymmetric jets in uniform crossflow.

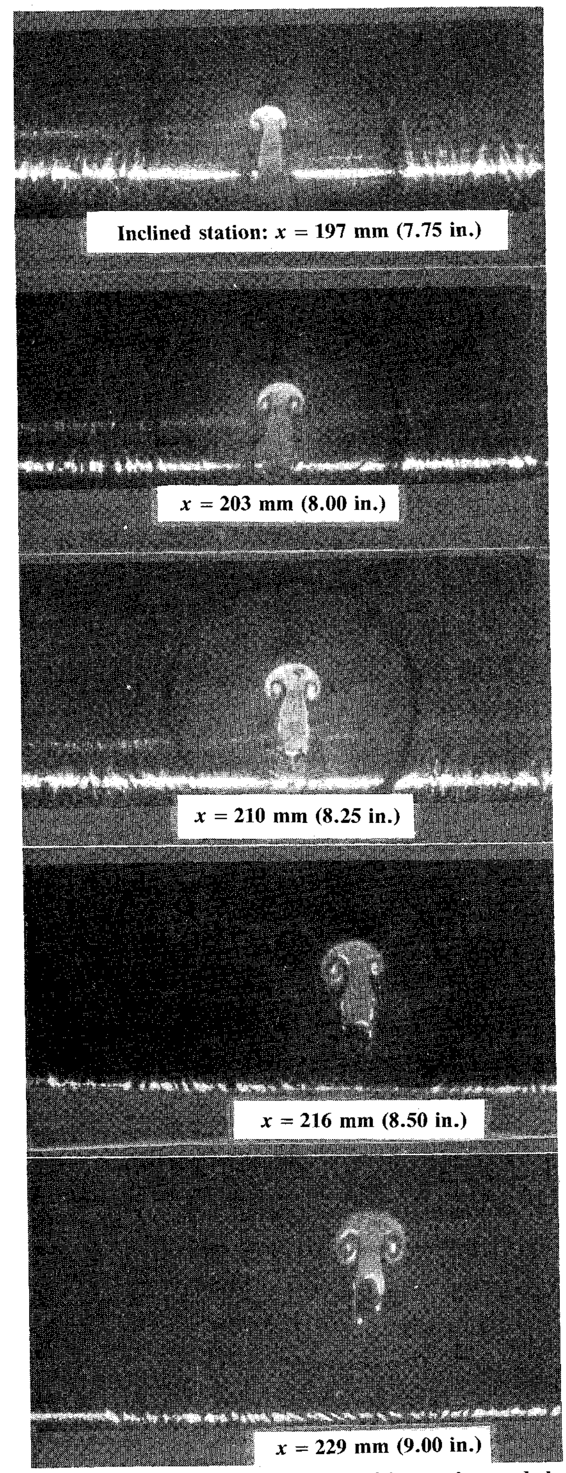


Fig. 10 Sectional view of the evolution of jet vortices and shearing on the jet side surface with 45 deg inclined viewing planes; model 3, $V_j/V_\infty = 2.0$, $\beta = 180$ deg, $Re_d = 470$.

Figure 8 shows the development of jet vortices for a teardrop-shaped jet (model 3) at various downstream distances. These photos are taken in a plane inclined 45 deg toward the oncoming flow using the laser fluorescent flow visualization technique. This figure shows how the jet fluid is deforming into the jet vortices and demonstrates the formation of a wake vortex near the flat-plate surface.

Based on evidence presented earlier, the explanation of how the flow developed from a symmetric jet to a nonsymmetric jet in crossflow is given in Fig. 9. With the aid of an imaginary plane (A), a pair of jet vortices and a pair of wake vortices can be seen. If the jet is symmetric, these two pairs of vortices would be symmetric, as illustrated in Fig. 9. With a nonsymmetric jet yawed to oncoming flow on the same plane (A), the wind-side jet vortex moves away from the base plate, and the lee-side jet vortex moves closer to the plate, which enhances the lee-side wake vortex and diminishes the wind-side wake vortex, as shown in Fig. 9. The pair of wake vortices are the product of induced flow by the counter-rotating jet vortices near the plate (laser-fluorscent photos have clearly shown this wake vortices formation). Figure 9 explains the sense of rotation for the wake vortices. Note here that the shearing to the jet fluid by the crossflow is quite different on the wind and the lee sides. This explains why there are three distinct vortices in the flowfield of Fig. 6.

For a teardrop-shaped cross-section jet positioned in the crossflow, with the narrow end forward ($\beta = 180$ deg), the observed flowfield is shown in Fig. 10. Under crossflow shearing action on the jet in a symmetric manner, vortices form around it that shed from the jet trailing end in a periodic but symmetric manner. Figure 10 shows cross sections of jet flow developing in parallel planes at 45 deg to the flat plate after the jet exit into the downstream.

B. Unsteady Jet Vortices

The existence of periodicity in the flow behind jets in crossflow has been reported by many investigators, for instance, Gordier.[13] McAllister[14] related the shedding frequencies of vortices due to crossflow and a circular jet, with the Strouhal number based on the jet diameter. Reilly[15] identified two eddy groups shed periodically from different regions of the jet.

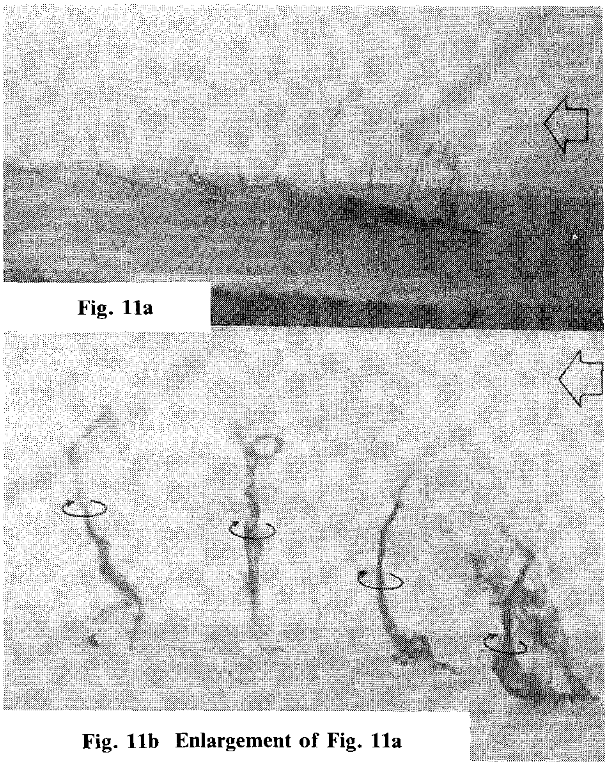


Fig. 11a

Fig. 11b Enlargement of Fig. 11a

Fig. 11 Periodically shed "spin-off" vortices in the downstream of jet model 3; $V_j/V_\infty = 3.3$, $\beta = 20$ deg, $Re_d = 240$.

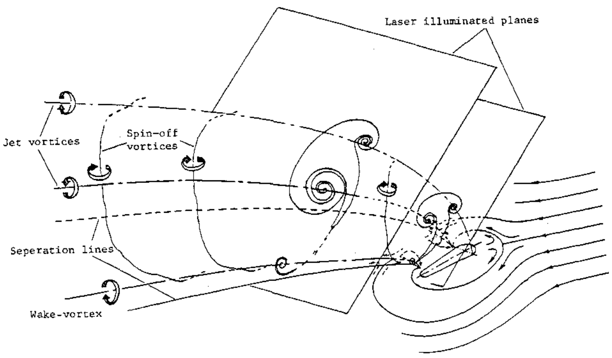


Fig. 12 Schematic of the flowfield of yawed jet in crossflow.

In the present nonsymmetric jet study, it was found that under certain flow conditions, especially at higher jet/freestream velocity ratios (which correspond to higher momentum ratios), periodic shedding vortices were observed in the downstream of asymmetric jets in the crossflow (Fig. 11). These periodic vortices were initiated on the jet wind side through shearing generated at the interface between the jet flow and the uniform crossflow.

These types of periodic vortices were observed primarily for jet models 2, 3, and 4 at relative yaw orientations of 5–30 deg to the tunnel freestream and with the jet/tunnel velocity ratios of $V_j/V_\infty > 3$. Figure 11 shows two typical photographs of the flow containing the periodic vortices. We have labeled these "spin-off" vortices, borrowing this terminology from our wing tip work,[2–4] because of similarities between these vortices and the spin-off vortices observed there. The spin-off vortices shared a common root, with their other ends entrained into the steady jet vortices. The strength and shedding frequency of these spin-off vortices appeared to increase with the jet velocity or its velocity ratio V_j/V_∞.

Boldman et al.[16] studied two-dimensional vortices shedding from a flat-plate blunt trailing edge with equal and unequal velocities applied on both sides of the plate. They found that symmetric and alternating von Kármán vortices appeared in the wake with equal velocities applied as expected. Decreasing the velocity on only one side of the plate, however, weakened that side of the vortices and, in the end, it was possible to eliminate altogether that side of the vortices.

This analysis could be used to explain the result of present observation of the "spin-off" vortices shedding from the wind side of jet only and with the same sense of rotation. Referring to Fig. 9, one would expect a higher shear on the wind side of the jet surface than on the lee side. This would be greater with a moderate increase in the jet velocity and with a moderately yawing angle β, which would provide a larger shearing surface. The shedding frequency appeared to be related to the jet/freestream velocity ratio (or momentum ratio) and the jet orientation. On the lee side of the jets, certain periodicity was present in the separated flow, which was either coupled with the vortices and the shedding frequencies or was influenced by it, since the frequency of the wake unsteadiness and shedding were the same.

The typical common root of the spin-off vortices is shown in various figures (see Figs. 3, 6, 9, and 11) and is referred to as the jet "wake vortex" in subsection A. Both laser-fluorescence and the regular dye indicated that the jet wake vortex initiated from the lee side of the jet and developed parallel to the jet vortices but close to the flat plate. The wake vortex was visible for moderate to higher jet flow rates only for nonsymmetric jet configurations.

When spin-off vortices were present, they appeared to roll around each other and the wake vortex. It should be noted here that many investigators[17–20] have observed similar rolling-up vortices in the wake flow structure behind a cylinder with its axis perpendicular to a flat plate in uniform crossflow. Taneda, in his observation,[17] explained that vortices from the end of a cylinder must rotate in an alternating sense and connect to similar strength, yet alternatively shed von Kármán vortices behind the cylinder. In our present observation, the wake vortex bundled with spin-off vortices were always rotating in the same sense as that of the wind-side jet vortex flow. Figure 12 shows a schematic of the various vortices and features of the asymmetric jets in a uniform flowfield.

To clarify the relationship between unsteadiness in the jet wake and the spin-off vortices, artificial periodic perturbations of the various frequencies were produced on the plate in lee side of the jets. Perturbations were created by angular oscillations of a cylindrical rod, at the tip of which a small flat-plate fin was installed. The 6- $\times$ 4-mm rectangular fin was positioned at the tip of the 3-mm-diam rod, and its presence in the wake, when not being rotated, did not influence the flow. As a result of the added periodicity, vortices similar to spin-off vortices were shed for asymmetric jets and for frequencies corresponding to the wake fluctuations. Figure 13 shows two flowfields, indicating that difference resulting from the added perturbations.

C. Pulsating Jet

In order to investigate the effect of jet flow periodicity on the resulting flowfield and especially on the various vortices in the downstream, pulsation was applied to the jet flow. A frequency generator and a solenoid valve were used to introduce pulsation in the jet. That is, the continuous jet was replaced by a periodically blown jet. The circular jet (model 1) and the teardrop-shaped jet (model 3) were subjected to variable-frequency pulsations.

At low frequencies, 1–4 Hz, the jet penetration into the crossflow increased significantly for both models 1 and 3. Vortex rings were produced by the pulsation at 1 Hz, which actually impinged on the tunnel top wall. The vortex rings were symmetric for the circular jet, but they had a complicated shape for the teardrop-shaped jet (model 3). The vortex rings

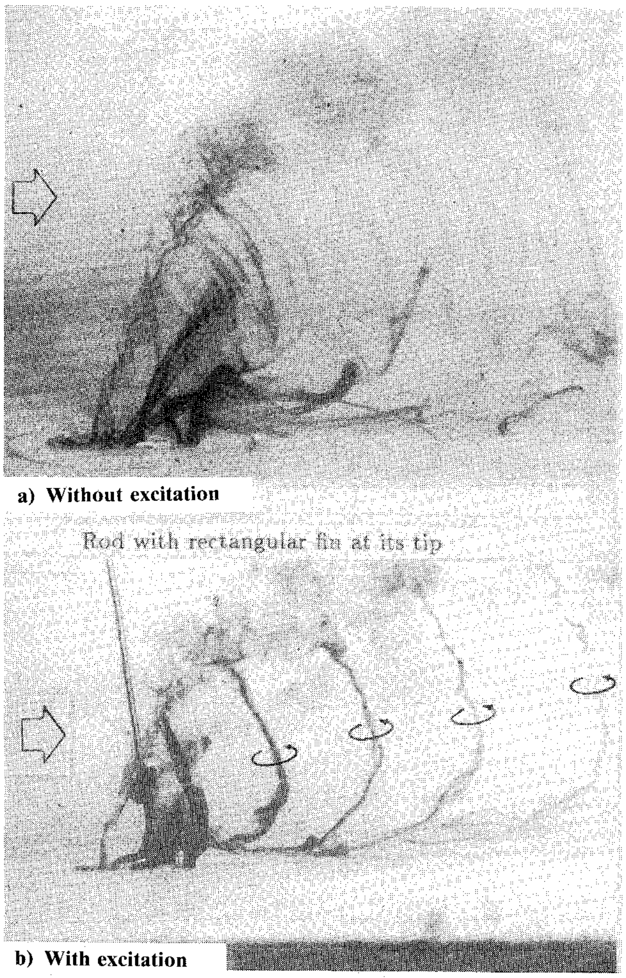


Fig. 13 Artificially induced spin-off vortices with oscillating fin behind the jet port; model 3, $V_j/V_\infty = 2.5$, $\beta = 10$ deg, $Re_d = 540$.

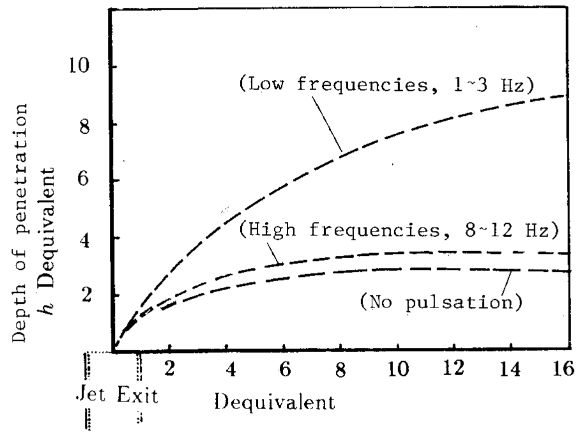


Fig. 14 Jet plume centerline trajectory with pulsating jet stream; model 3, $V_j/V_\infty = 4.7$, $\beta = 45$ deg, $Re_d = 470$.

of this model were also rotating around the axis of the jet along which they were moving into the crossflow. The increased penetration of the jet is believed to be due partially to the increased momentum in the jet pulses, since the mean flow rate was maintained constant.

As the frequency of the pulsation increased, the flowfield gradually returned to the normal pattern, with little difference from that of the steady jet, including its penetration height. At low pulsation frequencies, spin-off vortices were not observed for the teardrop-shaped jet, but they reappeared as the fre-

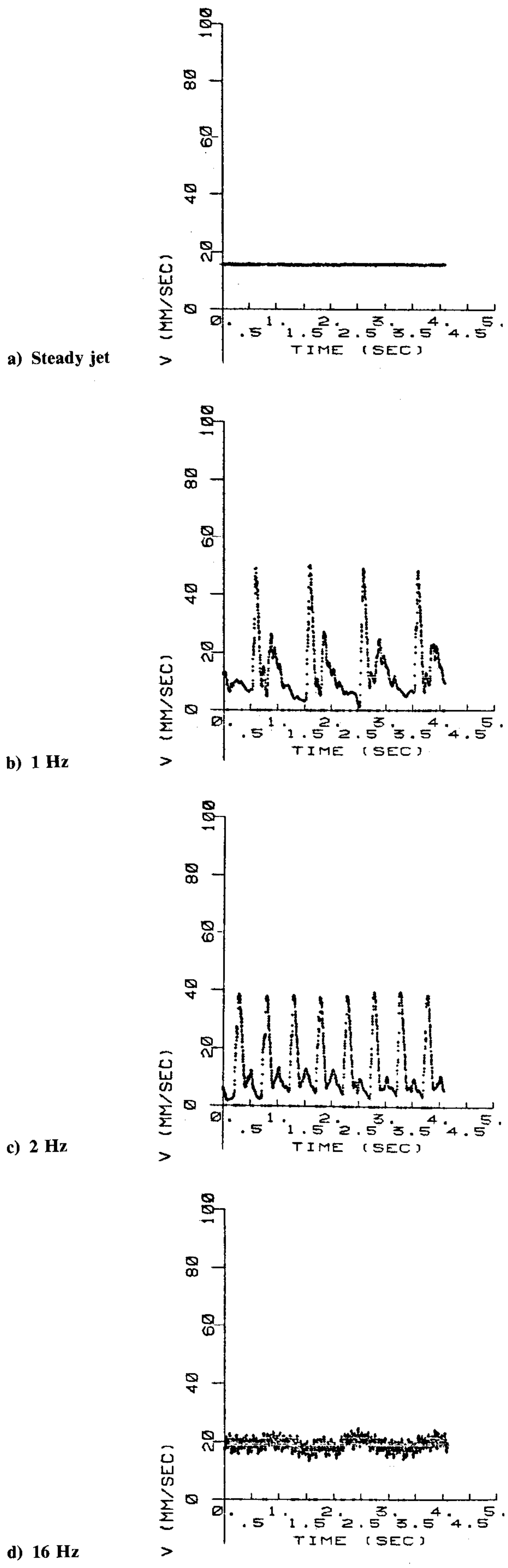


Fig. 15 Pulsating jet velocity as a function of time measured with hot film.

quency of pulsation increased beyond 16 Hz. A qualitative representation of the jet plume trajectory is shown in Fig. 14.

Measurement of three velocity components on the circular jet with pulsations was carried out. The measured time history of the velocity components with no pulsation is plotted on Fig. 15a. The same flow conditions with a pulsing jet in Fig. 15b shows velocity components with a pulsation frequency of 1 Hz. It clearly shows that the vertical component of velocity has a peak nearly three times larger. As the frequency of pulsation becomes 2 Hz, the peak of v velocity component is decreased to about 2.2 times that of the value of a continuous jet, as shown in Fig. 15c. For pulsation frequency as high as 16 Hz, Fig. 15d indicates that the peak of the v velocity component shows only small oscillations about the mean value. As can be seen in Fig. 15, peak velocities exist on these time histograms. These peaks are the major factors that cause the dramatic increase in depth of penetration. With experiments on the vortex ring formation at a tube opening with a piston moving upstream, Pullin[21] found that the strength of the vortex ring depends on the piston velocity and the piston diameter. In the present investigation, the time histogram of the velocity corresponds to the piston velocity profile. With the peak velocity varying in the jet supply tube, different strengths of vortex ring structures existed and this caused the difference in the depth of penetration of the jet. The peak velocity decreases as the pulsation frequency increases.

IV. Concluding Remarks

The flowfields of four different cross-section geometry jets in crossflow have been studied and compared with that of a circular cross-section jet. The jets had equal cross-section areas but different geometries. This qualitative study was conducted in a water tunnel for detail flowfield visualization. The flow around asymmetric jets in crossflow is very complex, and it contained many features significantly different from that of symmetric jets in crossflow. The flowfield has been reconstructed schematically, with flow physics discussed (Figs. 9 and 12).

Periodic vortices perpendicular to the crossflow were observed. These were termed "spin-off" vortices. Spin-off vortices were observed for asymmetric jets only, appeared only at jet velocity/freestream velocity ratios usually higher than 2.5 or 3, and were dependent on jet port shape and orientation.

Perturbations introduced into the wake of asymmetric jets produced the spin-off vortices artificially only when the frequency of the perturbations matched the separated wake flow fluctuations. The amplitude of the natural fluctuations was not large enough to cause the vorticity created as a result of shearing between the jet and the crossflow to shed under such circumstances.

A wake vortex, which was lying parallel and close to the floor plate, was observed for asymmetric jets. Once the spin-off vortices were present, they became twisted and linked near the flat plate and wrapped around the wake vortex.

The jet flow pulsated at low frequencies resulted in formation of vortex rings that penetrated into the main flow more deeply than the jet flow pulsated at high frequencies. Under this condition, no spin-off vortices could be produced. At higher pulsation frequencies ($f > 16$ Hz), the flowfields were similar to those of the steady jets, with increased mixing present at the jets boundary.

There appears to be significant similarities between jet flows and the flow around asymmetric cylindrical bodies in uniform crossflow.[20] Identification of these flow features is a first step toward full understanding, utilization, enhancement, or suppression of selected vortices in many fluid mechanics applications.

Acknowledgments

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